NAG Toolbox for MATLAB

e02rb

1 Purpose

e02rb evaluates a rational function at a user-supplied point, given the numerator and denominator coefficients.

2 Syntax

```
[ans, ifail] = e02rb(a, b, x, 'ia', ia, 'ib', ib)
```

3 Description

Given a real value x and the coefficients a_j, for $j = 0, \ldots, l$ and b_k, for $k = 0, \ldots, m$, e02rb evaluates the rational function

$$\frac{\sum_{j=0}^{l} a_j x^j}{\sum_{k=0}^{m} b_k x^k}.$$

using nested multiplication (see Conte and de Boor 1965).

A particular use of e02rb is to compute values of the Padé approximants determined by e02ra.

4 References

Conte S D and de Boor C 1965 *Elementary Numerical Analysis* McGraw–Hill

Peters G and Wilkinson J H 1971 Practical problems arising in the solution of polynomial equations *J. Inst. Maths. Applics.* **8** 16–35

5 Parameters

5.1 Compulsory Input Parameters

1: **a(ia) – double array**

$\mathbf{a}(j+1)$, for $j = 1, 2, \ldots, l+1$, must contain the value of the coefficient a_j in the numerator of the rational function.

2: **b(ib) – double array**

$\mathbf{b}(k+1)$, for $k = 1, 2, \ldots, m+1$, must contain the value of the coefficient b_k in the denominator of the rational function.

Constraint: if $\mathbf{ib} = 1$, $\mathbf{b}(1) \neq 0$.

3: **x – double scalar**

The point x at which the rational function is to be evaluated.

5.2 Optional Input Parameters

1: **ia – int32 scalar**

Default: The dimension of the array **a**.

the value of $l + 1$, where l is the degree of the numerator.

Constraint: **ia** ≥ 1.

2: **ib – int32 scalar**

Default: The dimension of the array **b**.

the value of $m + 1$, where m is the degree of the denominator.

Constraint: **ib** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ans – double scalar**

The result of evaluating the rational function at the given point x.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The rational function is being evaluated at or near a pole.

ifail $= 2$

On entry, **ia** < 1,
or **ib** < 1,
or **b**$(1) = 0.0$ when **ib** $= 1$ (so the denominator is identically zero).

7 Accuracy

A running error analysis for polynomial evaluation by nested multiplication using the recurrence suggested by Kahan (see Peters and Wilkinson 1971) is used to detect whether you are attempting to evaluate the approximant at or near a pole.

8 Further Comments

The time taken is approximately proportional to $l + m$.

9 Example

```
a = [1;
     0.5000000000000011;
     0.1071428571428577;
     0.011904761904762;
     0.0005952380952381046];
b = [1;
     -0.4999999999999989;
     0.1071428571428566;
     -0.01190476190476179;
```

```
     0.0005952380952380861];
x = 0.1;
[ans, ifail] = e02rb(a, b, x)
```

```
ans =
    1.1052
ifail =
           0
```